Using ADFs for Inconsistency-Tolerant Query Answering with Existential Rules

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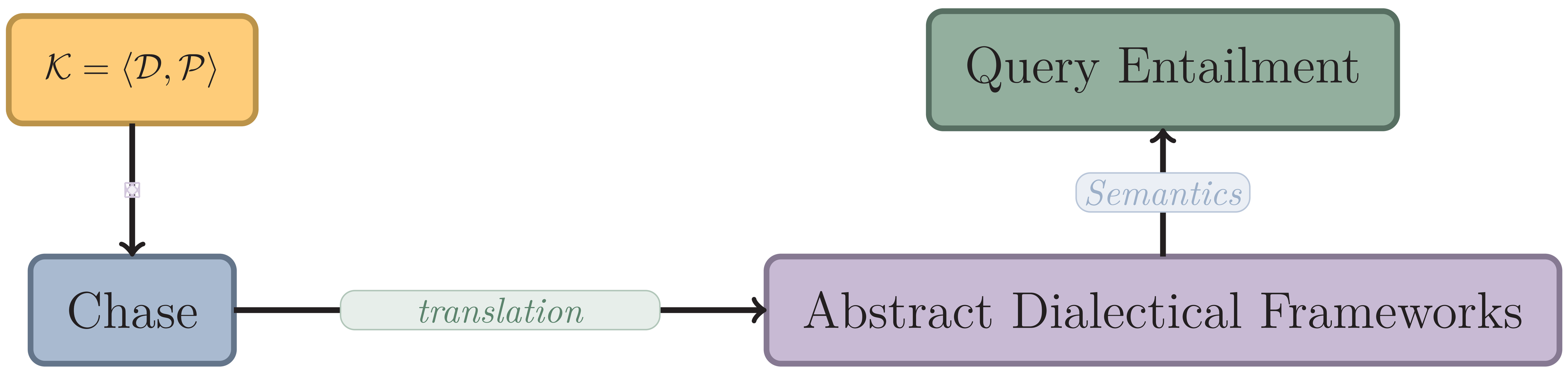
Filling the Gap: Proof-Based Explanations for Query Answers Under Inconsistency-Tolerant Semantics

Let $\mathcal{K} = \langle \mathcal{D}, \mathcal{P} \rangle$ be KB such that $\mathcal{D}= \{$ Goldfish(a), location(a, b), Sky(b) $\}$
$\mathcal{P}=\{$ Fish(x)$\wedge Bird(x) \rightarrow \bot, \quad Goldfish(x) \rightarrow Mobile(x) \wedge Fish(x),$
$location(x, y) \wedge Sky(y) \rightarrow \exists z.Mobile(x) \wedge fliesTowards(x, z), \quad fliesTowards(x, y) \rightarrow Bird(x)\}$

- Why is "a" a bird?
- Because: location (a, b), Sky (b)
- **Why is "a" a bird?**

Outline

Generate proof explanations for query answers in an inconsistent ontology using *abstract dialectical frameworks*



Abstract Dialectical Frameworks

An ADF is a tuple $F = (A, L, C)$ where: 1) A is a finite set of nodes (arguments, statements) 2) $L \subseteq A \times A$ is a set of links 3) $C = \{\varphi_a\}_{a \in A}$ is a collection of propositional formulas (acceptance conditions).

Semantics: Methods used to clarify the acceptance of arguments in ADFs.

- A *three-valued interpretation*: $v : A \rightarrow \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$
- *Characteristic Operator*: $\Gamma_D(v) = v'$ s.t.

$$v'(a) = \begin{cases} \mathbf{t} & \text{if } \varphi_a^v \text{ is irrefutable,} \\ \mathbf{f} & \text{if } \varphi_a^v \text{ is unsatisfiable,} \\ \mathbf{u} & \text{otherwise.} \end{cases}$$

where $\varphi_a^v = \varphi_a[p/\top : v(p) = \mathbf{t}][p/\bot : v(p) = \mathbf{f}]$

- $v \in admissible(D)$ if $v \leq_i \Gamma_D(v)$
- $v \in preferred(D)$ if v is $\leq_i$-maximal admissible

Induced ADF from a Given KB

Given a KB $\mathcal{K} = \langle \mathcal{D}, \mathcal{P} \rangle$. The induced ADF $D(\mathcal{K}) = (S_\mathcal{K}, L_\mathcal{K}, C_\mathcal{K})$ is:

1. $S_\mathcal{K} = \mathsf{chase}(\mathcal{K})$
2. for each $B \in S_\mathcal{K}$, $\varphi_B = \varphi_B^1 \wedge \varphi_B^2 \wedge \varphi_B^3$:
   $\varphi_B^1 = \bigwedge \{ \bigwedge_{i=1}^n C_i \rightarrow D \mid B \wedge \bigwedge_{i=1}^n C_i \rightarrow D \in \mathcal{P}_\mathcal{D}^{ch}, C_i \in S_\mathcal{K}, D \in S_\mathcal{K} \}$
   $\varphi_B^2 = \bigvee \{ \bigwedge_{i=1}^n A_i \mid \bigwedge_{i=1}^n A_i \rightarrow B \in \mathcal{P}_\mathcal{D}^{ch}, A_i \in S_\mathcal{K} \}$
   $\varphi_B^3 = \bigwedge \{ \neg E \mid B \wedge E \rightarrow \bot \in \mathcal{P}_\mathcal{D}^{ch}, E \in S_\mathcal{K} \}$

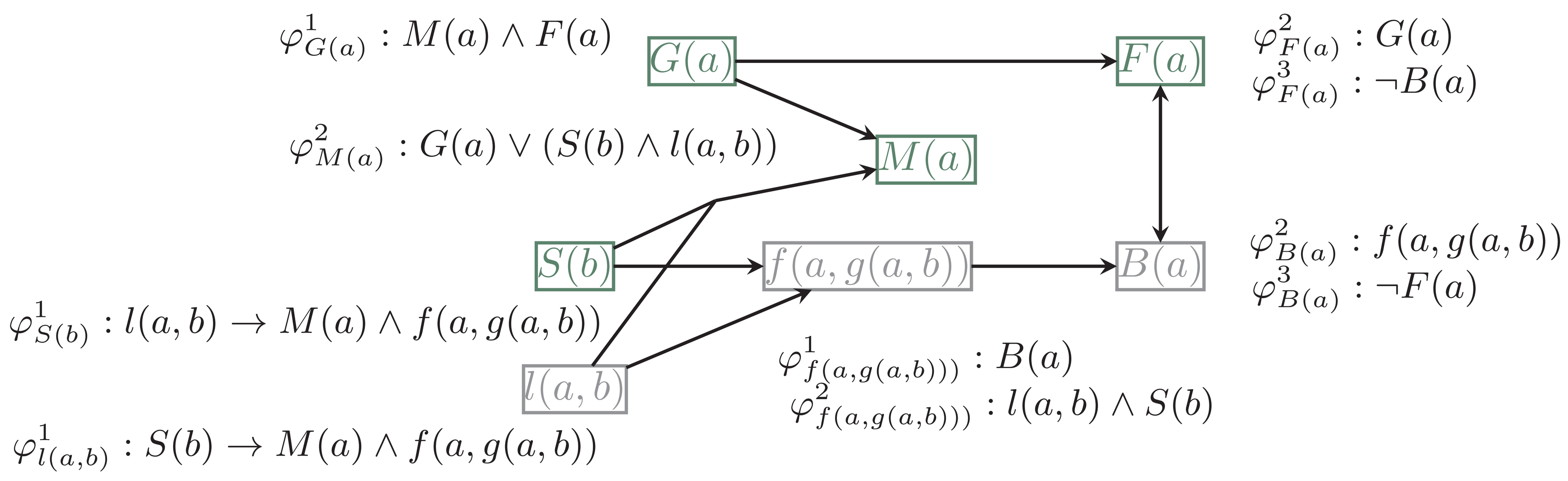
Minmax-Preferred Semantics for induced ADF

An interpretation v is *Minmax-preferred interpretation* (for $D(\mathcal{K})$), if

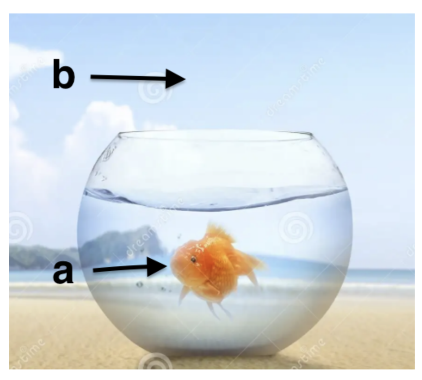
1. $v \in prf(D)$,
2. $v^\mathbf{t}|_\mathcal{D}$ is *maximal* among all preferred interpretations of $D(\mathcal{K})$, i.e., if $w \in prf(D)$, then $v^\mathbf{t}|_\mathcal{D} \not\subset w^\mathbf{t}|_\mathcal{D}$,
3. $v^\mathbf{t}$ is *minimal* among all $w \in prf(D)$.

Query Answering Explanation via ADFs

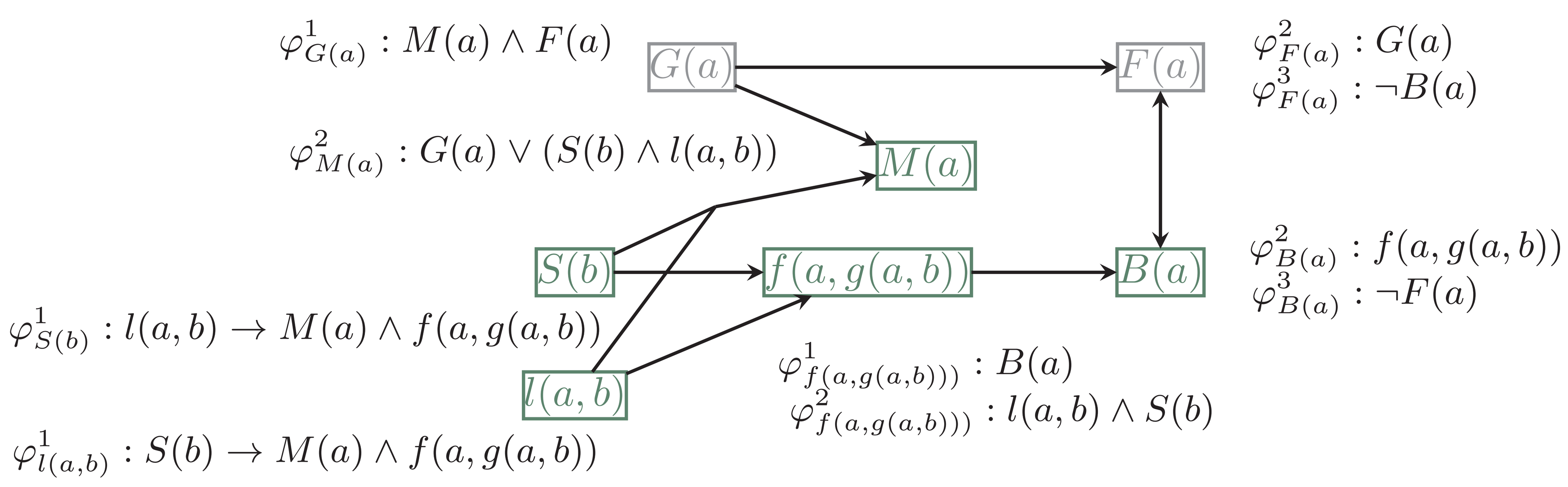
$\mathcal{K} \models_{\texttt{brave}} F(a)$: Why is a a fish?



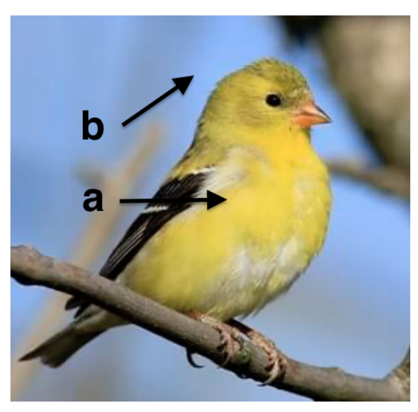
- $v = \{G(a), M(a), F(a), \neg B(a), \neg f(a, g(a, b)), S(b), \neg l(a, b)\}$



$\mathcal{K} \models_{\texttt{brave}} B(a)$: Why is a a bird?



- $v' = \{\neg G(a), \neg M(a), \neg F(a), B(a), f(a, g(a, b)), S(b), l(a, b)\}$



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